A Framework for Level Set-Based Topology Optimization with Constrained Shape Updates

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Outline

1. Motivation: topology optimization of parts in an assembly

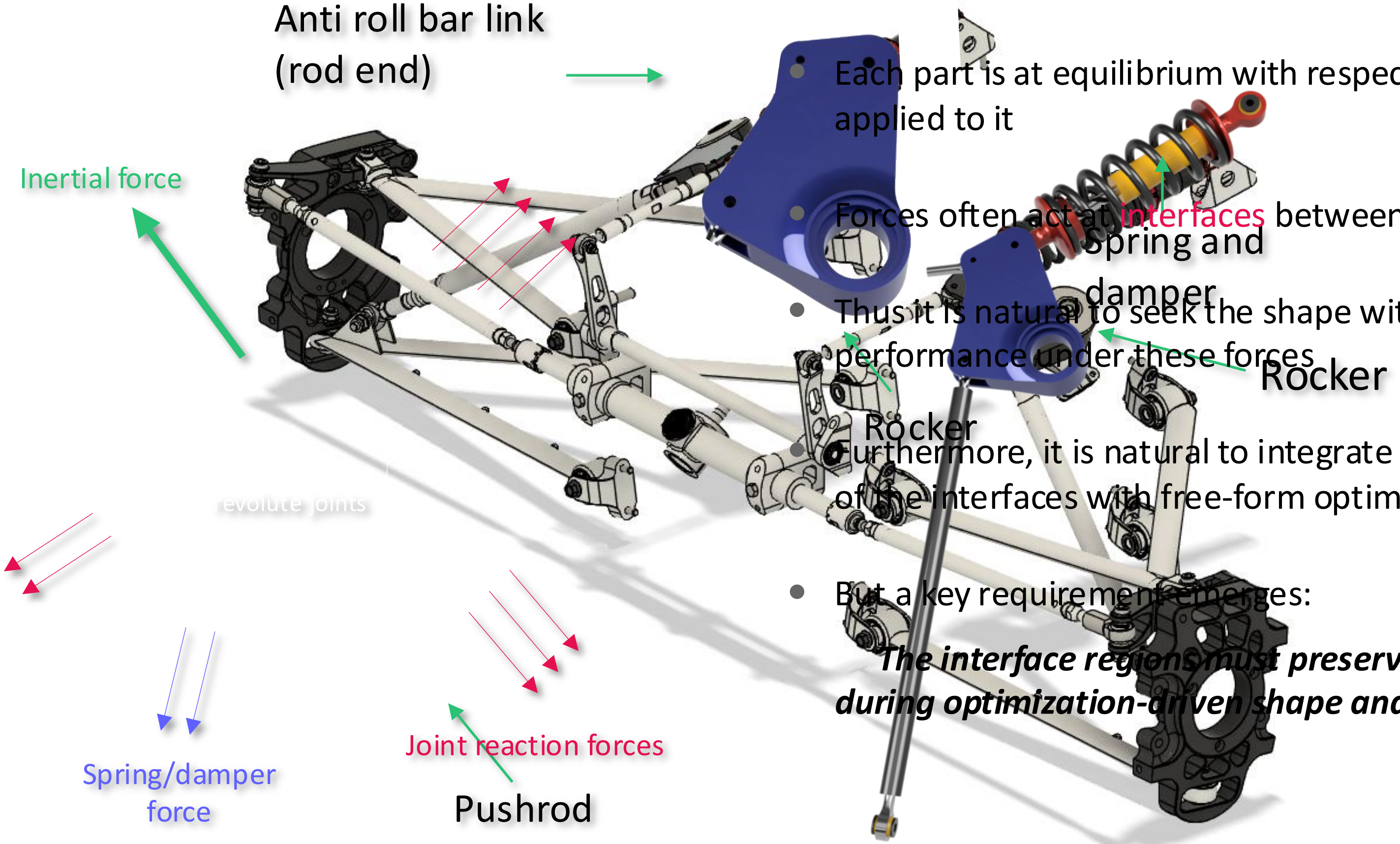
2. Level set updates that preserve interfaces between parts in assemblies

3. Simultaneous level set-based topology optimization of parts and their interfaces

Topology Optimization of Parts in Assemblies

Credit: Audi

Topology Optimization of Parts in Assemblies



- Each part is at equilibrium with respect to the forces applied to it
- Forces often act at interfaces between parts
- Thus it is natural to seek the shape with optimal performance under these forces
- Furthermore, it is natural to integrate the optimization of the interfaces with free-form optimization
- But a key requirement emerges:

  The interface regions must preserve their "nature" during optimization-driven shape and interface changes

Some Related Work

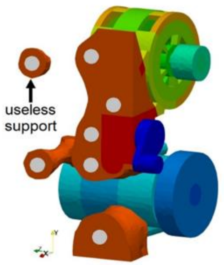
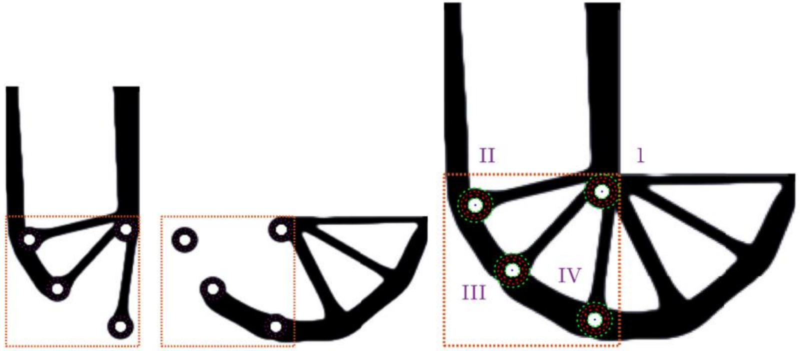


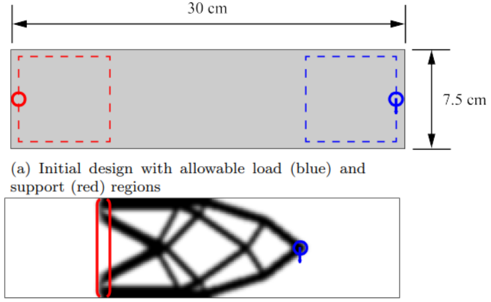
Figure 9: Coupled support locations and structure optimization

Rakotondrainibe, Allaire and Orval [2020]



(A) Left part (B) Right part (C) Assembly

Ambrozkiewicz and Kriegesman [2020]



(a) Initial design with allowable load (blue) and support (red) regions

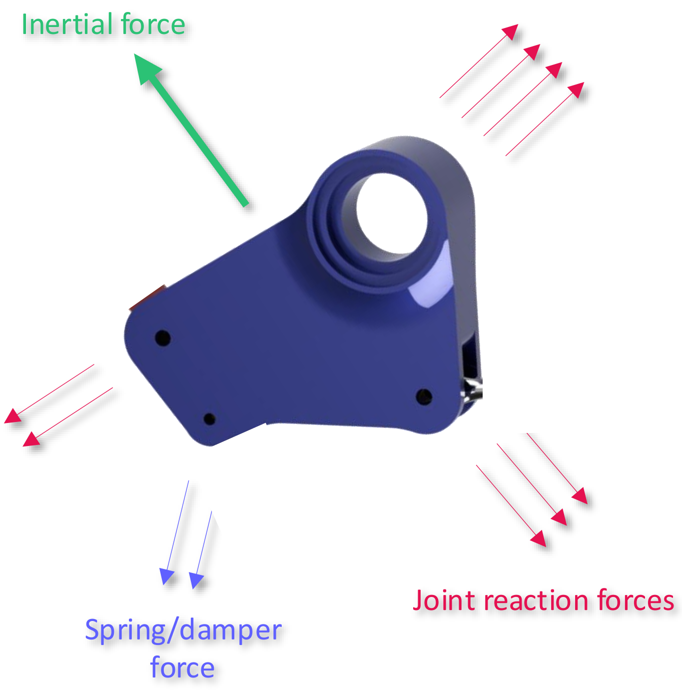
(b) Final optimized design

Alacoque and James [2021]

- Optimized shape and support locations
- Optimized shape and joint positions
- Optimized shape, loads and support locations

Our Approach

- Enhancement of level set-based topology optimization
- Provides the ability to preserve the nature of assembly interfaces
- Gives parametric control (translation, rotation and scaling) over any interface region
  - Loads and supports
  - Joint interfaces
- Allows simultaneous optimization of free geometry and interface parameters



A Constrained Topology Optimization Problem

minimize	$\mathcal{J}(\Omega)$
subject to	$\partial\Omega$ contains interfaces of the specified nature
over	all admissible shapes Ω and all allowed interface parameters

Methodology

1. A level set representation for a shape that includes interface data:

   $$\Omega := \{F(x) \leq 0\} \text{ with } \partial\Omega \text{ containing interfaces of specified types}$$

2. An interface-preserving level set update procedure:

   $$F \longrightarrow F_{new}$$
   $$\Omega_{new} := \{F_{new}(x) \leq 0\}$$
   $$\partial\Omega_{new} \text{ contains new interfaces of the same types}$$

3. A method to choose such an update based on sensitivity analysis, ensuring that we obtain a descent direction for the optimization objective $\mathcal{J}$, namely:

   $$\mathcal{J}(\Omega_{new}) < \mathcal{J}(\Omega)$$

Level Set Representation with Interfaces

Level set function F

- As usual

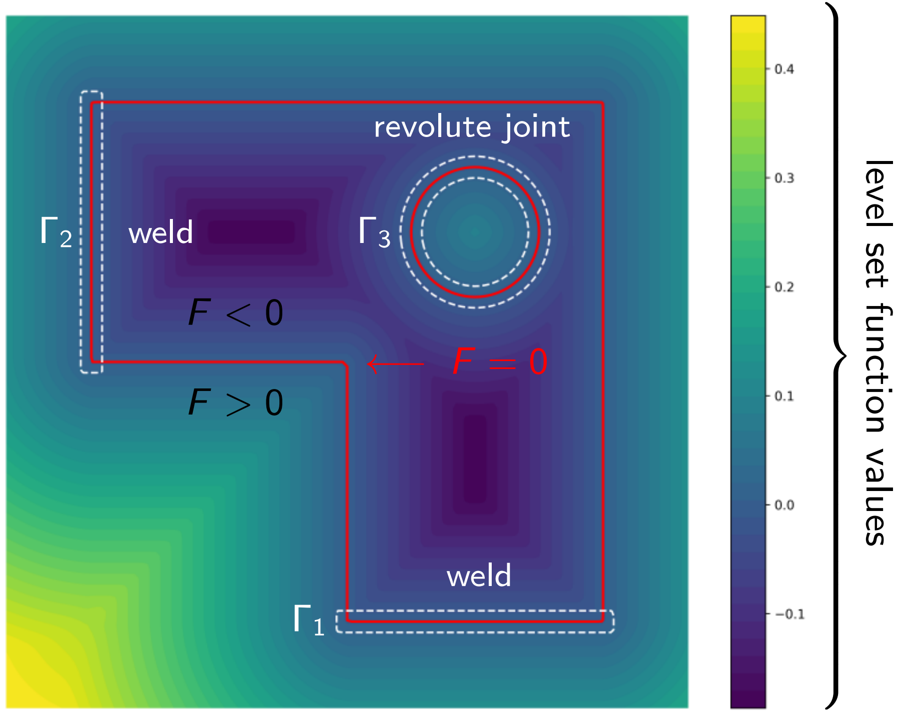
Interfaces Γ are subsets of $\partial\Omega$ with

- Type

  Γ.type = weld, revolute joint, etc.

- Simple parametric geometry

  Γ.params = location, orientation, size, etc.



Conventional Level Set Updates

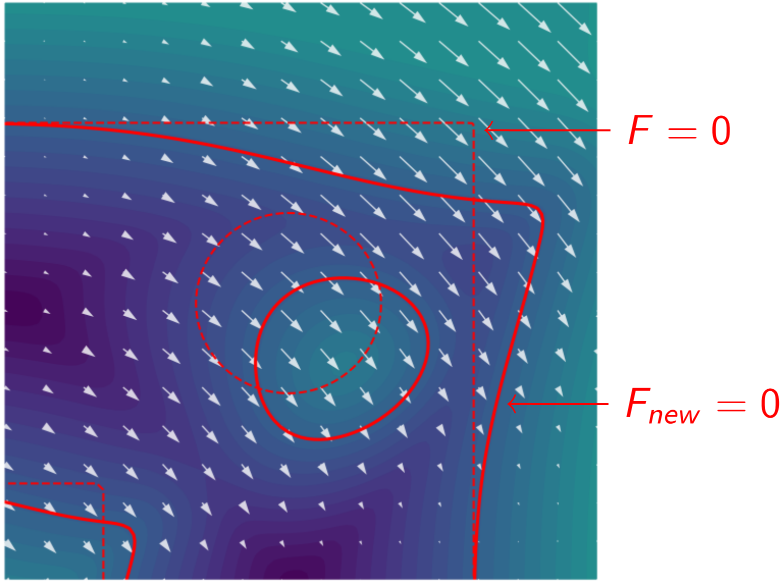
To update a level set function F

- We prescribe a velocity field Θ on a domain surrounding the zero level set
- Then we solve the transport equation in a prescribed time interval $[0, t_{new}]$

$$\frac{\partial F_t}{\partial t} + \langle \nabla F_t, \Theta \rangle = 0$$
$$F_0 = F$$

- The updated level set function is

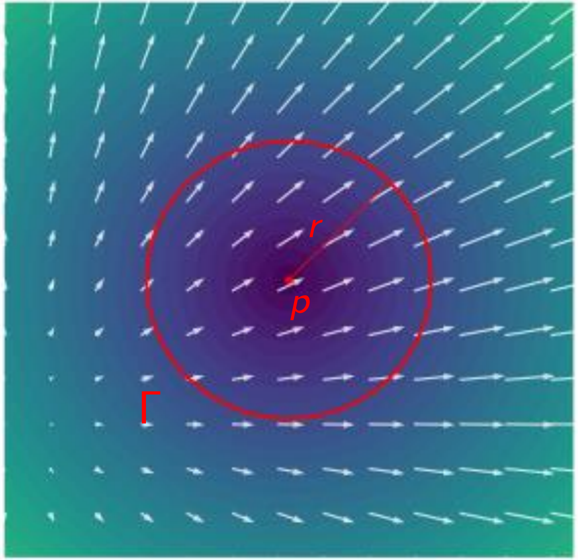
$$F_{new} := F_{t_{new}}$$



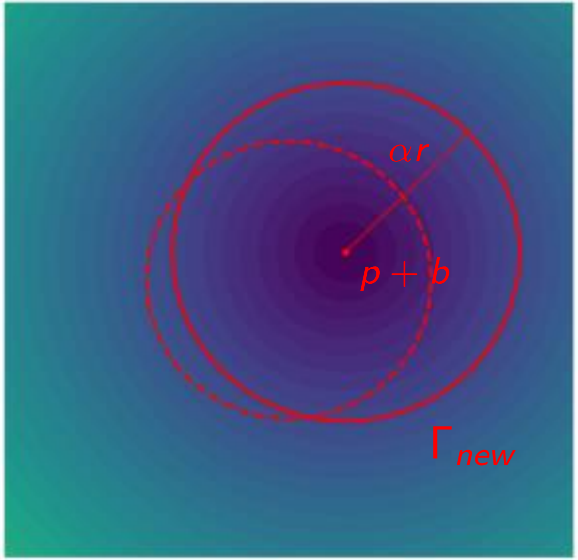
Note: circular region is distorted!

Allowed Motions at the Interfaces

The velocity Θ must be such that the nature of the interface is preserved by transport



Γ.type = revolute joint
Γ.params.radius $= r$
Γ.params.centre $= p$



$[\Gamma_{new}]$.type = revolute joint
$[\Gamma_{new}]$.params.radius $= \alpha r$
$[\Gamma_{new}]$.params.centre $= p + b$

Interface-Preserving Motions

- Motions that are combinations of **translation**, **rotation**, and **orthotropic scaling** will preserve the nature of the interface

$$x(t) := \text{Translate}(\text{Rotate}(\text{Scale}(x)))$$

***Note order of operations!

Translate: time t, direction $a \in \mathbb{R}^3$

Rotate: time t, angular velocity $B \in Antisym(\mathbb{R}^{3\times 3})$

Scale: time t, scale factor $c \in \mathbb{R}^3$

- Velocity fields that produce these motions have the form

$$\Theta^{a,B,c}_{allowed}(x,t) := a + B(x - at) + \text{Rotate}\Big(\text{diag}(c)\big(\text{Rotate}^{-1}(x - at)\big)\Big)$$

Interface-Preserving Level Set Updates

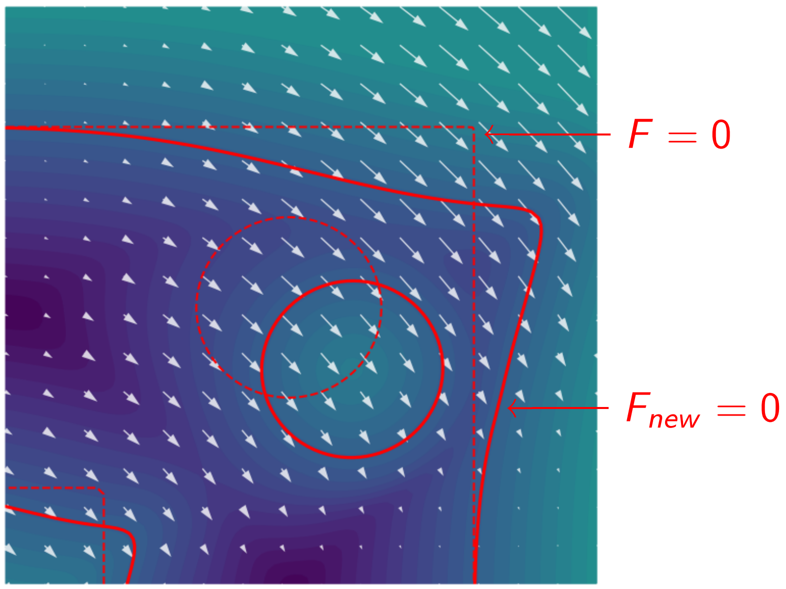
So: To update a level set function F in an interface-preserving way

- Impose the constraint

$$\Theta = \Theta_{allowed}^{a,B,c}$$

near interfaces, with appropriate a, B, c

- Solve the transport equation for F_{new} using the constrained vector field



Note: circular region remains circular!

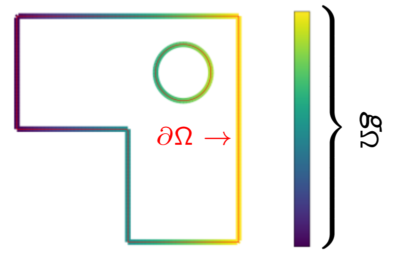
Choice of Update Velocity

Requirement: Θ must be a descent direction for the optimization objective

- Recall that we have

$$\mathcal{J}(\Omega_{new}) \approx \mathcal{J}(\Omega) + t_{new} \int_{\partial\Omega} g_\Omega \, \Theta^\perp$$

where g_Ω is the shape gradient



g_Ω as a scalar function on $\partial\Omega$

- Therefore we need a vector field satisfying

(1) $$\int_{\partial\Omega} g_\Omega \, \Theta^\perp < 0$$

(2) $\Theta = \Theta^{a,B,c}_{allowed}$ near interfaces, where a, B, c are to be determined by (1)

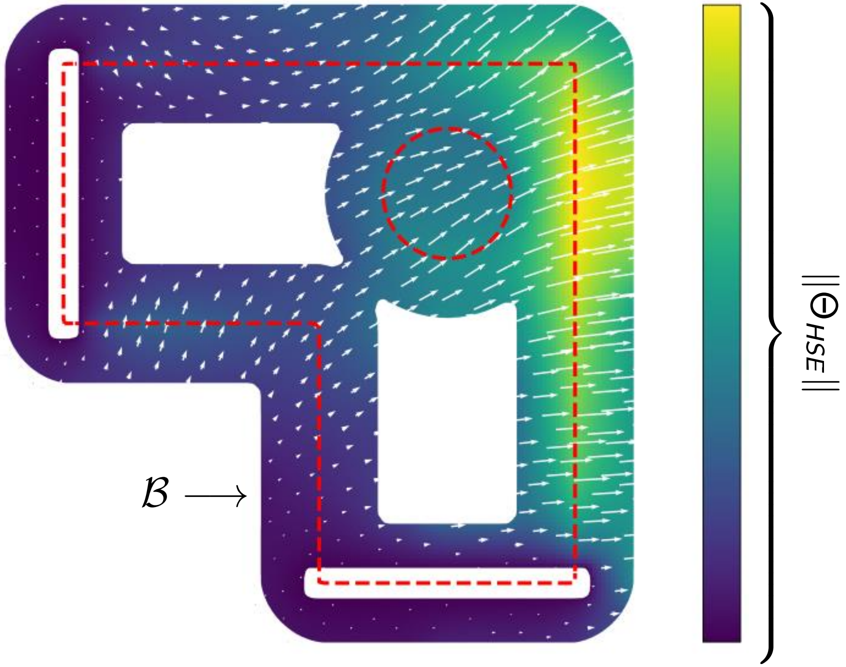
Hilbert Space Velocity Extension Procedure

Reminder: Descent directions can be found using the following procedure

- Consider velocity fields defined on a narrow band $\mathcal{B}$ of $\partial\Omega$
- Let $\mathbf{H}$ be a smoothing inner product on the Hilbert space of vector fields on $\mathcal{B}$
- Solve the variational problem

$$\Theta_{HSE} := \arg\min_{\Theta} \frac{1}{2}\mathbf{H}(\Theta, \Theta) - \int_{\partial\Omega} g_\Omega \, \Theta^\perp$$

- Then $-\Theta_{HSE}$ is a descent direction



HSE Means Solving a PDE

The smoothing inner product often has the form

$$\mathbf{H}(\Theta, \Theta) := \int_{\mathcal{B}} \left(\Theta \cdot \Theta + \gamma \nabla\Theta : \nabla\Theta\right)$$

γ a smoothing parameter

Therefore Θ_{HSE} satisfies the PDE

$\Theta - \gamma\Delta\Theta = g_{\Omega}$	in $\mathcal{B}$
Essential & natural BCs	on $\partial\mathcal{B}$

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where we now view g_{Ω} as a distributional source term supported on $\partial\Omega$

Imposing Interface-Preservation Constraints

How: We obtain our desired extension velocity Θ_{CHSE} in two steps.

Step 1: We modify the HSE procedure by adding allowed velocity constraints

- Allowed velocities satisfy $\Theta^{a,B,c}_{allowed}(x, t = 0) = a + Bx + \mathrm{diag}(c)x$ for some a, B, c
- Thus we let Θ_* solve the **constrained variational problem**

$$\begin{aligned} &\text{minimize} && \frac{1}{2}\mathbf{H}(\Theta,\Theta) - \int_{\partial\Omega} g_\Omega\, \Theta^\perp \\ &\text{subject to} && \Theta = a + Bx + \mathrm{diag}(c)x \;\text{ near the interface } \Gamma \\ &\text{over} && \Theta \text{ and all } \underbrace{a, B, c}_{\text{determined by the variational problem!}} \end{aligned}$$

Imposing Interface-Preservation Constraints

Step 2: We create the correct time-dependence near the interfaces

- Define the extension velocity $\Theta_{CHSE}(x,t)$ by "blending" it with the known time-dependent velocity near the interface

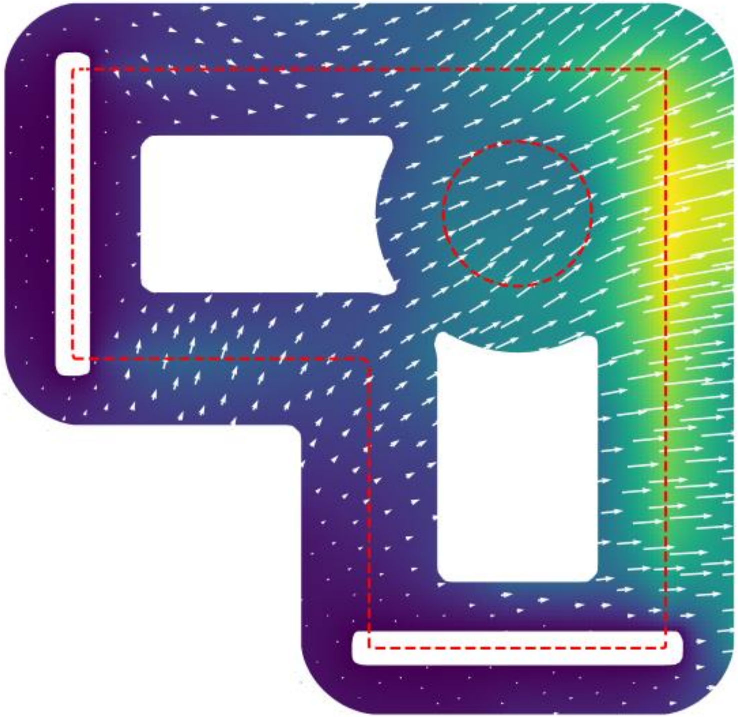
$$\Theta_{CHSE}(x,t) := \big(1-\eta(x)\big)\,\Theta_*(x) + \eta(x)\;\Theta^{a,B,c}_{allowed}(x,t)$$

↑ cut-off function for Γ

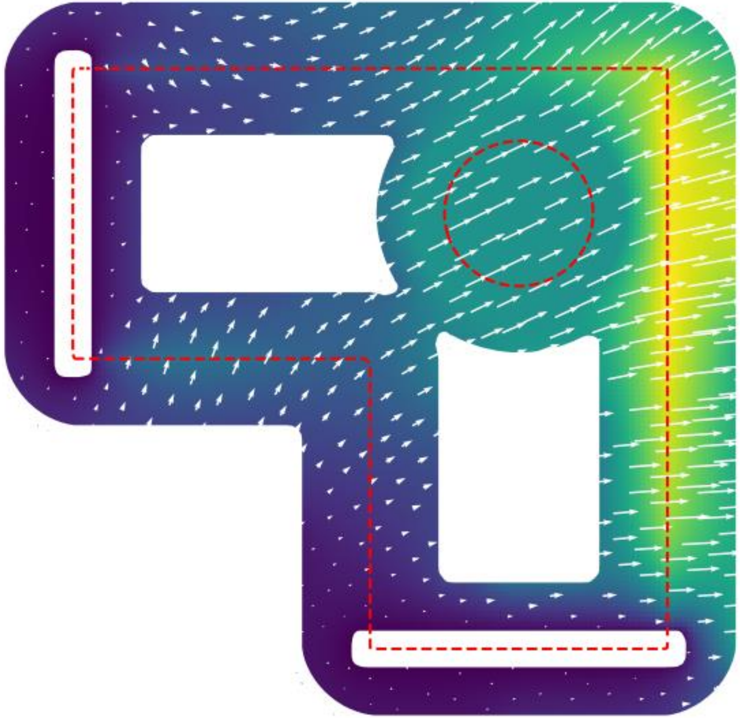
Result: By construction $-\Theta_{CHSE}$ is spacetime-continuous, satisfies the allowed velocity constraints, and is a descent direction for the optimization objective

Example

Conventional Θ_{HSE}

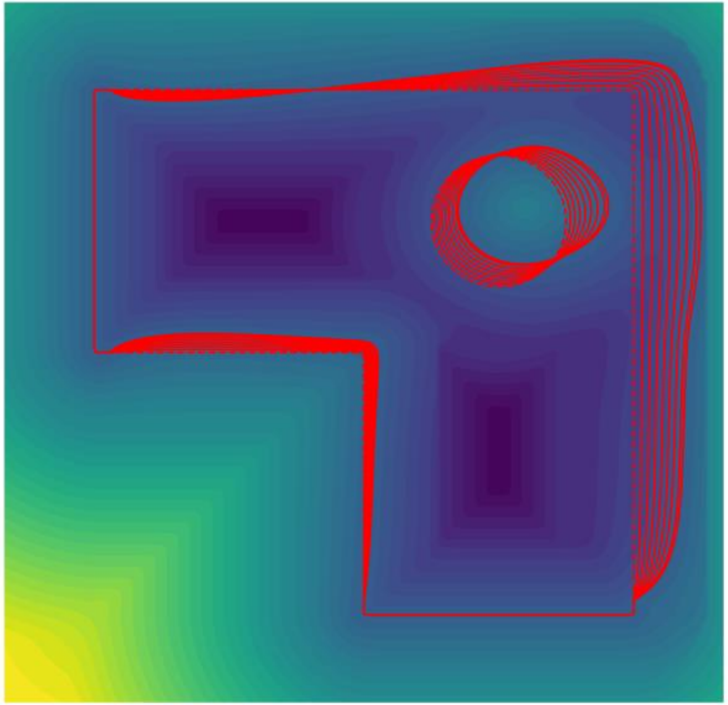


Θ_{CHSE} with translation constraint imposed near the revolute joint

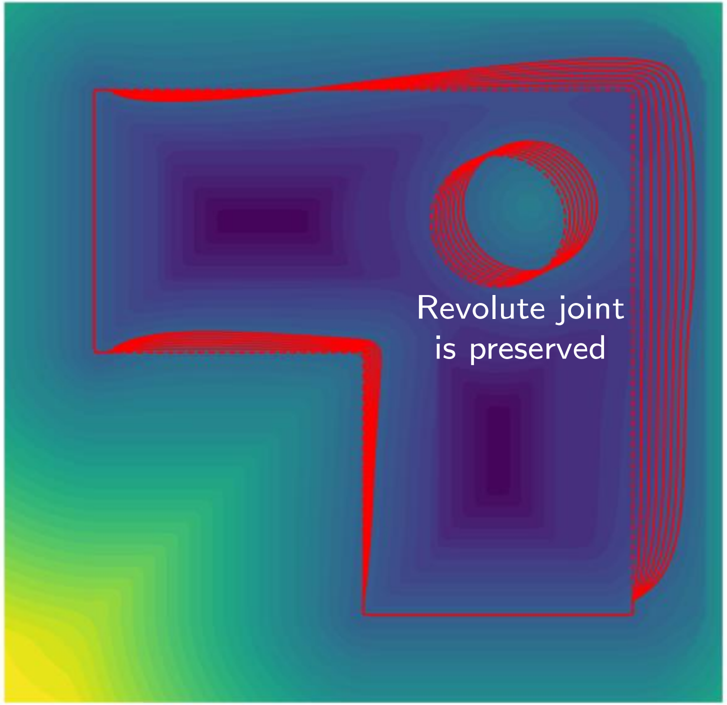


Example

Transport with respect to Θ_{HSE}



Transport with respect to Θ_{CHSE}



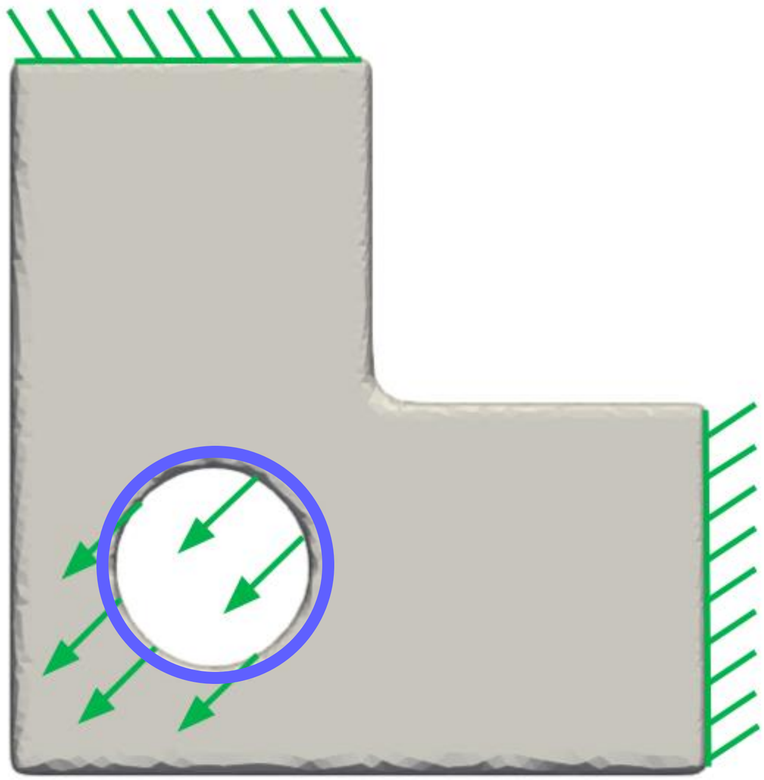
Results — Bracket

Consider an L-bracket with an aperture (blue) for a revolute joint

We pose a simplified*** load scenario:

- Fixed boundary conditions on the top and right surfaces
- A constant load applied to the surface of the aperture

*** A single-part load scenario representative of the type of loading the part might experience within an assembly, but much simpler

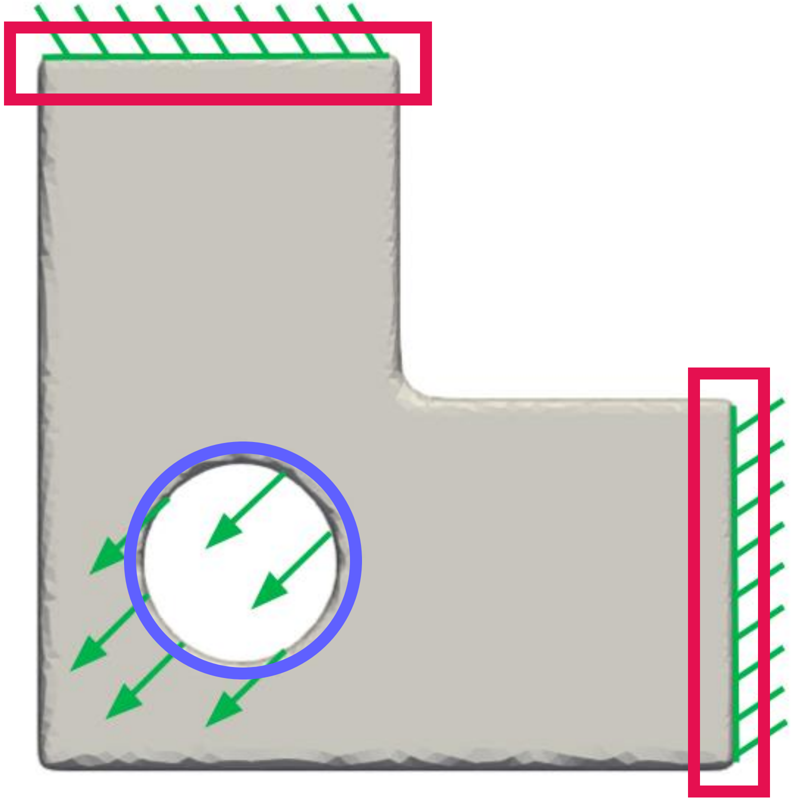


Results — Bracket

And we pose a design optimization problem for this load scenario:

Minimize compliance with a volume fraction constraint and the restrictions

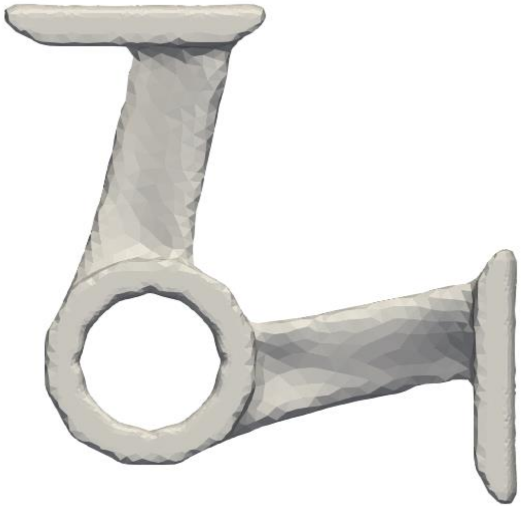
- The fixed interfaces (red) must not move or change
- The aperture must remain cylindrical

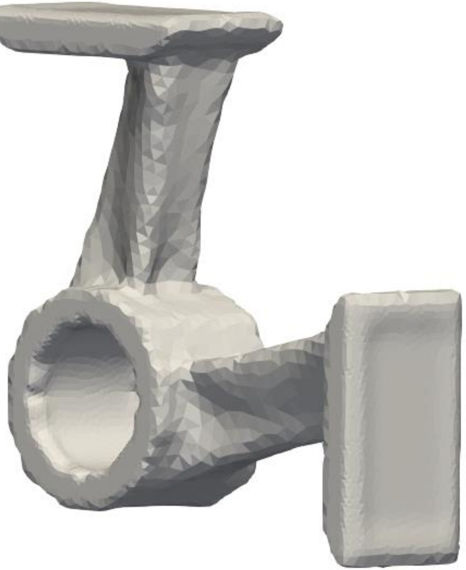


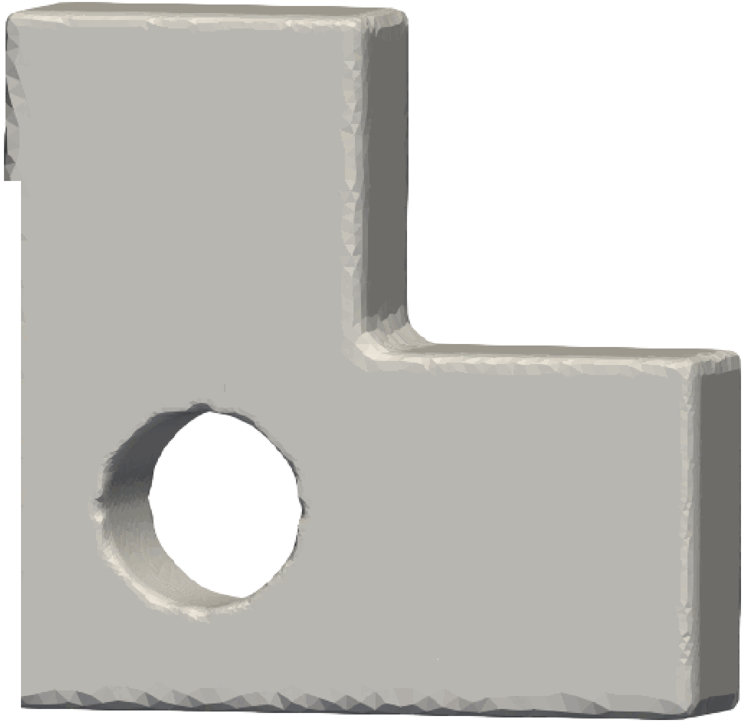
Results — Bracket

Case 1: Frozen Constraint
Here, the aperture is not allowed to move

Final shape:



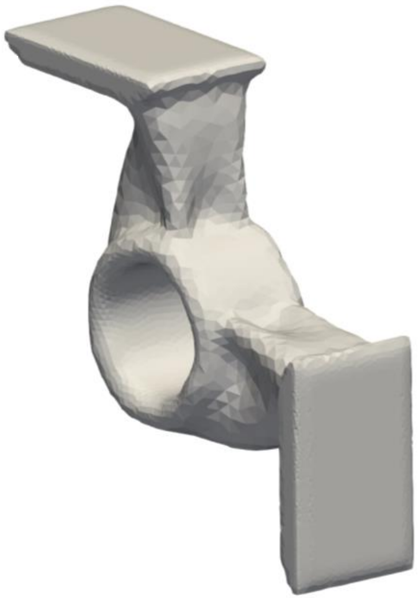


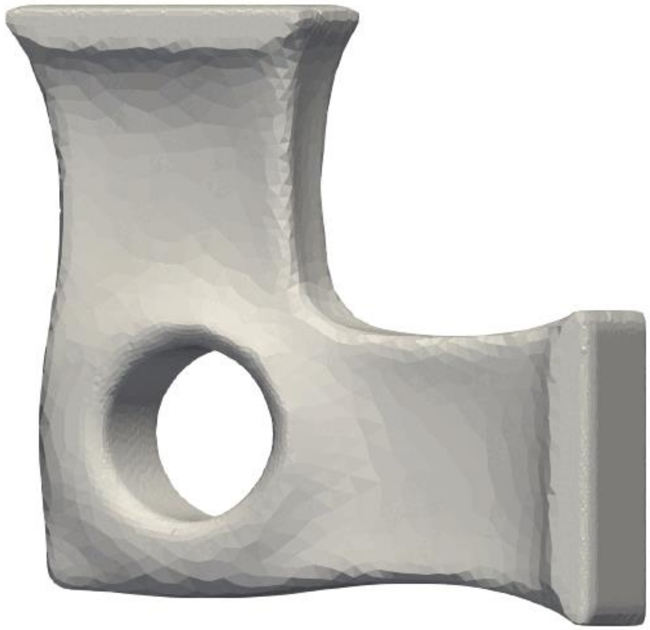


Results — Bracket

Case 2: Translation Constraint
Here, the aperture is allowed to translate freely

Final shape:





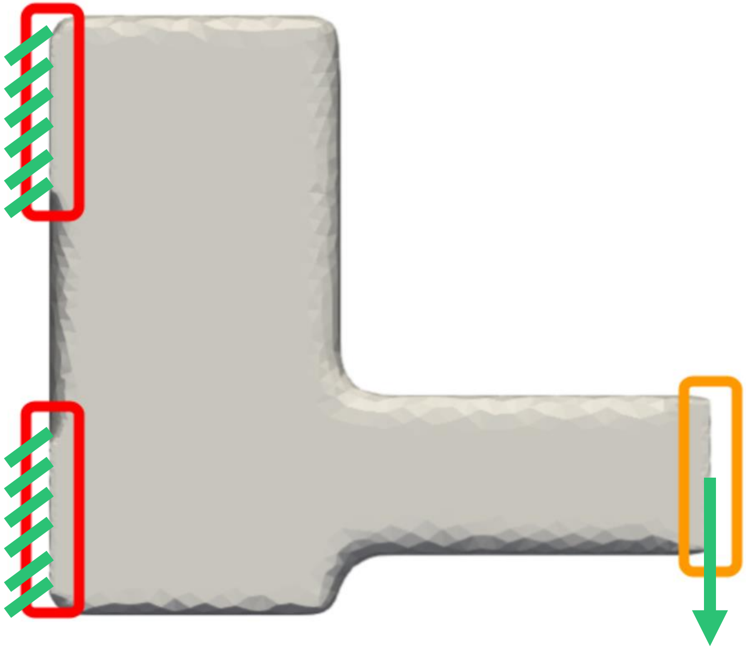
Results — Cantilever

We now pose a new asymmetric cantilever load scenario

And a new design optimization problem:

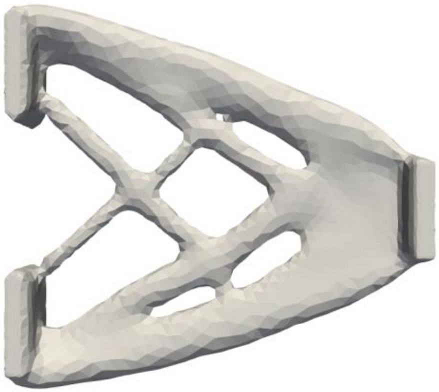
Minimize compliance with a volume fraction constraint and the restrictions

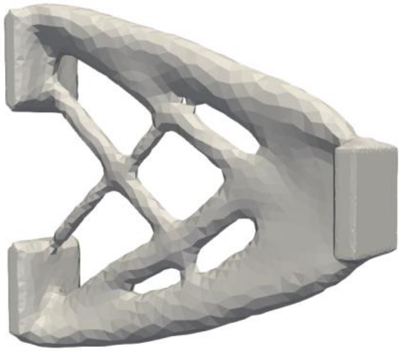
- The red interfaces must not move
- The orange box is allowed to translate vertically

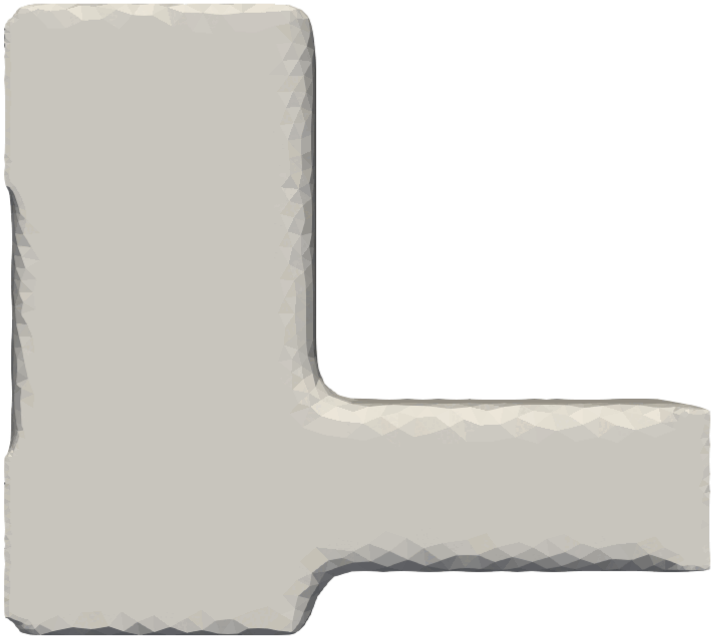


Results — Cantilever

The load regions translates such that the result approaches a symmetric Michell Truss







Future work

- More complicated parametric geometry and variations of interfaces
- Full-assembly simulation with shape-dependent reaction forces at the interfaces
- Simultaneous optimization of multiple parts in the assembly

Conclusion

- New shape update framework for level set-based topology optimization, meant for optimization of parts within assemblies

- We use it to give parametric control (translation, rotation and scaling) over any interface region:
  - Loads and supports
  - Joint interfaces